Secondary reggeons in perturbative QCD

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Abstract. Using the Kirschner–Lipatov equation as a basis, we study high energy elastic quark–quark amplitudes, which correspond to secondary reggeons and pion exchange in the double-logarithmic approximation of perturbative QCD.

The high energy behavior of elastic scattering amplitudes is described usually in terms of t-channel reggeon pole (and cut) exchange; that is, in terms of the singularities in the complex moment j-plane (see, for example, [1]). The rightmost singularity, the pomeron, has the quantum numbers of the vacuum and is placed at $j \approx 1$. In the framework of perturbative QCD the corresponding singularity is formed by the two (reggeized) gluon cut and its properties are given by the BFKL equation [2,3] (see [4, 5] for more details). The secondary reggeon contribution corresponds to an amplitude with quark–antiquark pair t-channel exchange. The leading double-logarithmic (DL) asymptotics of this amplitude was calculated in perturbative QCD by Kirschner–Lipatov [6,7]. It appears that the quark–antiquark cut occurs at

$$j = \omega_0(t) = \sqrt{2C_F \alpha_S / \pi} \qquad (1)$$

for the positive signature amplitude. Here α_S is the QCD coupling, $C_F = (N_c^2 - 1)/2N_c$ and $N_c = 3$ is the number of colors.

In the $N_c \to \infty$ limit, the positive (even) and negative (odd) signature DL amplitudes are degenerate and both are given by the sum of the ladder-type Feynman diagrams. For finite $N_c = 3$, there appears a non-ladder DL contribution to the odd signature amplitude, which shifts the position of the singularity in j-plane. However this non-ladder DL contribution is not large; numerically $\omega_0^{(-)} \simeq 1.04 \omega_0^{(+)}$, while for the reasonable value of $\alpha_S = 0.25$ the effective intercept $\omega_0^{(+)} \simeq 0.46$. This is close to the phenomenological intercepts of the ω, ρ (odd signature) and A_2 (even signature) trajectories – $\alpha_R(0) \approx 0.5$ which are approximately degenerate. Besides the ω, ρ, f and A_2 Regge poles there are the contributions with other t-channel quantum numbers, corresponding to π or A_1 (unnatural P-parity) exchange. In the present paper we discuss the properties of such contributions in the framework of the DL approximation of perturbative QCD.

Recall that the DL Kirschner–Lipatov quark–quark amplitude [6] may be written as the Born amplitude A_B (shown by continuous lines in Fig. 1) multiplied by a DL function $F_{DL}(\alpha_S \ln^2 s/|t|)$ which collects the loop contributions caused by the additional gluons. For example, for the simplest color singlet (in the t-channel) amplitude, the function F_{DL} reads

$$F_{DL}^+ = \frac{I_1(2n)}{n}, \quad n = \sqrt{\frac{C_F \alpha_S(t)}{2\pi}} \ln \frac{s}{|t|}. \qquad (2)$$

Thus

$$A^+(s,t) = A_B \frac{I_1(2n)}{n}, \qquad (3)$$

where

$$A_B = -\frac{4}{9} \cdot 4\pi\alpha_S.$$

The factor 4/9 comes from averaging over the colors of the initial quarks. Since the Bessel function $I_1(2n) \propto e^{2n}$ at large n, the high energy ($s^{1/2}$) asymptotics of the amplitude (3) and (2) takes the form $A^+ \propto s^{2n} = s^{\omega_0^{(+)}}$.

Note that to DL accuracy, the additional gluons (shown in Fig. 1b by the dashed lines) are emitted just by the color charge of the quark and do not change either the spin structure of the original Born amplitude nor the isospin of the t-channel $q\bar{q}$-pair. Therefore, to select the contribution with known quantum numbers in the t-channel, it is enough to use the Fierz relation

$$\gamma_\mu \times \gamma_\mu|_{s\text{-channel}} \qquad (4)$$
$$\to 1 \times 1 - \frac{1}{2}\gamma_\nu \times \gamma_\nu + \frac{1}{2}i\gamma_\nu\gamma^5 \times i\gamma_\nu\gamma^5 - \gamma^5 \times \gamma^5|_{t\text{-channel}}.$$

As is seen from (4), there is no tensor ($\sigma_{\nu\nu'}$) component in the t-channel expansion of the Born amplitude. From the phenomenological viewpoint we have to consider the A_2 trajectory as the even signature component of the amplitude generated by the vector (γ_ν) t-channel current.

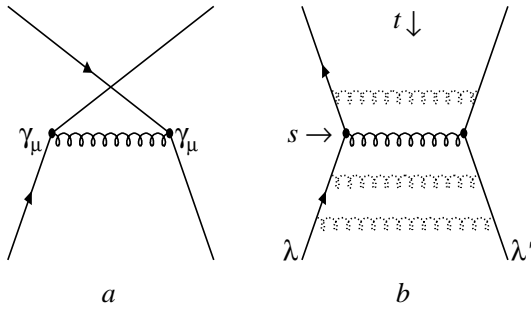**Fig. 1. a** The quark–quark and **b** quark–antiquark Born amplitudes. Higher loop corrections are shown **b** by *dotted lines*

Fig. 2. a The first loop $q\bar{q}$ amplitude and **b** the pseudoscalar (t-channel) component of the amplitude

The ρ and ω trajectories originate from the same vector (γ_ν) current but with odd signature. At the Born level, the relation (4) means that the amplitudes which correspond to natural and unnatural t-channel parities, that is γ_ν and $\gamma_\nu\gamma_5$ or 1 and γ_5 terms in (4), are degenerate. Of course, besides this amplitude, we have the interaction of quarks mediated by t-channel two (and more) gluon exchange, that is by pomeron (and pomeron cuts). Moreover the degeneracy between the vector and axial (corresponding to the $\gamma_\nu\gamma_5$ term in (4)) amplitudes may be spoiled by higher order loop corrections coming from the region of rather large distances (small quark transverse momenta $k_t \leq 1\,\text{GeV}$), where the effects of confinement are not negligible. This point was discussed in [8]. Contrary to the pure "soft" Regge phenomenological approach (where the intercept of the A_1 trajectory $\alpha_{A_1}(0) \leq 0$ [9] is lower than that for the ρ trajectory $\alpha_\rho(0) \simeq 0.5$) at large scales, the effective (isovector) intercept in the axial ($\gamma_\nu\gamma_5$) channel is expected to be a little larger than that in the vector channel [8], due to the s-channel pion contribution coming from the boundary between the perturbative and $k_t \leq 1\,\text{GeV}$ confinement region. Analogously, the contribution from the soft momentum range is expected to enlarge the effective intercept in the pion channel in comparison with that of the scalar isovector exchange (like it was for the axial (A_1) channel [8]).

Now let us consider the amplitude A_5 with pion quantum numbers in t-channel. This amplitude originates from the last (γ_5) term on the r.h.s. of (4). To avoid the admixture of the t-channel gluon state, we choose here the isovector component[1]. In the Born approximation the high energy spin-flip amplitude is

$$A_5^{\text{B}}\left(\lambda = \frac{1}{2}, \lambda' = -\frac{1}{2}; -\frac{1}{2}, \frac{1}{2}\right) \propto \alpha_s \frac{t}{s}. \qquad (5)$$

At high orders of α_S the amplitude A_5^{B} is multiplied by the same DL function $F_{\text{DL}}^{(+)}$ (2) for even signature exchange (or by the corresponding function $F_{\text{DL}}^{(-)}$ for odd signature exchange). Indeed let us consider the first loop diagram, Fig. 2a. With the help of Fierz identity (4), we replace the

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[1] It is known that due to the γ_5-anomaly in singlet channel, an admixture of the two-gluon state alters strongly the mass and other properties of the η'-meson
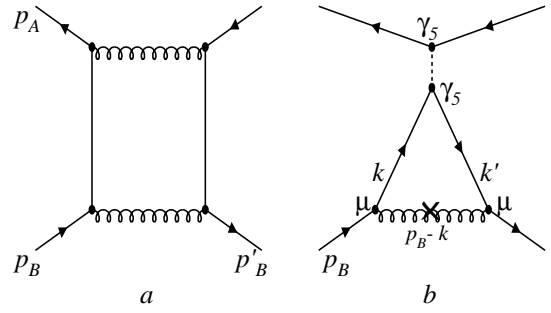
$\gamma_\mu \cdot \gamma_\mu$ vertices of the upper gluon in Fig. 2a by the effective γ_5 vertex in Fig. 2b in order to select the contribution with pion quantum numbers in the t-channel.

It is convenient to use the light-cone Sudakov variables

$$k_\mu = -\alpha p_{A\mu} + \beta p_{B\mu} + k_{\mu t}, \quad \text{d}^4 k_\mu = \frac{s}{2}\text{d}\alpha\text{d}\beta\text{d}^2 k_t,$$

$$s \simeq 2 p_A p_B, \qquad (6)$$

and to close the contour of the α-integral on the lower gluon pole $1/(p_B - k)^2$. Then the loop integral reads[2]

$$C_F \frac{g^2}{16\pi^3} \int \frac{\text{d}^2 k_t}{k^4} \frac{s\,\text{d}\beta}{s\beta - k_t^2}$$
$$\times \sum_{\mu=1,2} \bar{u}(p_B', \lambda')\gamma_\mu^\perp \hat{k}\gamma_5 \hat{k}' \gamma_\mu^\perp u(p_B, \lambda)$$
$$\simeq C_F \frac{\alpha_S}{2\pi} \int \frac{\text{d}\beta}{\beta} \frac{\text{d}k_t^2}{k_t^2} \bar{u}\gamma_5 u, \qquad (7)$$

where within the DL approximation we assume $k^2 \simeq k_t^2$ and $k_t \gg q_t = (p_B - p_B')_t$; therefore $k_t' \approx k_t$ and $(k'k) \simeq k^2$. In addition we account for the fact that the s-channel gluon $(p_B - k)$ is on-mass-shell and has only two transverse polarizations $\mu = 1, 2$ (see [10,6] for more details).

The interval of the $\text{d}\beta$ integration is restricted by the same DL constraint $\beta \gg k_t^2/s$ as that of the main (vector or axial in the t-channel) component of the quark–quark amplitude [6,10]. Thus the first loop correction takes the form

$$\frac{1}{2}\frac{C_F\alpha_S}{2\pi}\ln^2\frac{s}{|t|} = \frac{1}{2}n^2, \qquad (8)$$

which is equal exactly to the corresponding term in the expansion of the DL function F_{DL} (2). The same is valid for the DL contribution from higher loops. As was shown in [6], each higher DL correction does not depend on (or alter) the spin structure of the quark–quark amplitude.

There is an analogous situation for the scalar component (the first term in (4)). Within the DL approximation the scalar and pseudoscalar exchange are degenerate, just as are the vector and axial components, discussed above.

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[2] We consider here the ladder-type loop only as the nonladder DL correction comes from the pure classical current emission and does not depend on the spin structure of the amplitude at all

Thus at small distances the quark–quark QCD interaction, mediated by the $q\bar{q}$-pair (t-channel) exchange, contains the contributions coming from the vector, axial, scalar and pseudoscalar terms in the Born amplitude. Within the DL approximation these contributions never mix with each other. The higher α_S order DL loop corrections simply multiply each term of the Born amplitude by the same function $F_{DL}(\alpha_S^{1/2} \ln s/|t|)$; F_{DL}^+ (or F_{DL}^-) depending on the even (or odd) signature of the amplitude. The "vector" and "axial" components conserve the s-channel quark helicities, while the "scalar" and "pseudoscalar" components flip the helicities of the incoming quarks. However these spin-flip amplitudes are suppressed at high energies by one power of s; the pion-like, pseudoscalar exchange contains an extra factor $|t|/s$ in comparison with the "axial" contribution.

From the phenomenological viewpoint the "axial" (A_1-like) component describes the spin–spin correlation, that is the difference of the non-flip amplitudes with different helicities of the incoming quark. It may be responsible for the low-x behavior of the non-singlet polarized structure function g_1 and other spin–spin effects in deep inelastic processes.

Contrary to the "soft" Regge phenomenology (where the intercepts $\alpha_{A_1}(0) \le 0$, $\alpha_\pi(0) \simeq 0$ of the A_1 and π trajectories are smaller than the intercepts $\alpha_R(0) \simeq 0.5$ of natural parity ω, ρ, A_2 trajectories) the perturbative QCD DL amplitudes with natural and unnatural parities are degenerate; the vector (γ_ν) term is degenerate with the axial ($\gamma_\nu \gamma_5$) one, and the scalar term with the pseudoscalar contribution. Therefore at large scales (i.e. small distances) the high energy spin–spin correlation occurs mainly due to the perturbative QCD interaction; the non-perturbative A_1 exchange dies out with energy.

On the other hand the effective intercept, ω_π, of the perturbative QCD amplitude with pion t-channel quantum numbers $\omega_\pi = \omega_0 - 1$ is much smaller than $\alpha_\pi(0) \simeq 0$. So, for a not too large scale, or $|t|$, the spin-flip (unnatural parity) quark–quark interaction should be described mainly by the non-perturbative pion Regge trajectory. The pion-like perturbative exchange amplitude may reveal itself in deep inelastic events with the leading final neutron (target proton $\to$ neutron charge exchange) or in the polarization properties of a ρ-meson produced (with a rapidity gap) in a large momentum transferred t-domain, like in the process

$$\gamma + p \to \rho^o + X$$

studied at HERA by the ZEUS collaboration [11]. An admixture of pion-like exchange may help to explain the rather large (up to (10–20)%) double-flip contribution observed in the data [11].

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